Maths

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Maths Lead

At the heart of learning maths

At Longlands, we believe that every child has the potential to be successful in maths lessons. Maths in for everyone! We do not believe that children should be limited by prior attainment. Developing a positive attitude to this subject is essential. Teachers promote children's enjoyment of maths and provide opportunities for them to build a conceptual understanding before applying their knowledge to everyday problems and challenges. To do this we have adopted the maths teaching for mastery approach.

Our curriculum:

·builds on prior learning;

· has clear purpose and context where appropriate;

·exposes mathematical structures through the use of manipulatives and representations to help children understand key concepts;

·helps children to see generalisations by emphasising patterns;

·knowledge and skills and are taught systematically with children being expected to use them in different contexts within the maths lessons and also other subjects such as graphing for science work; daily minute maths tests used for children to practice recall of number facts and incidental telling time.

- lessons are structured differently according to the nature of what is being taught but a ping pong approach is common to all lessons because we can structure children's learning more effectively, using small steps so as many children as possible can keep-up with the learning in the lesson; we know from research (Ebbinghaus, 1885) that only 33% children remember what has been taught after one day, so we revisit ideas over a series of lessons;
- children rehearse their mathematics in every lesson using a range of strategies. These will differ from one lesson to the next but are likely to include: repetition of stem sentences; paired talk; individual, paired and group tasks; quick response methods sometimes with the use of whiteboards; pictorial and symbolic written recording in books and on whiteboards; children's tasks incorporate 'intelligent practice' to help them move on to the next stages of their learning.

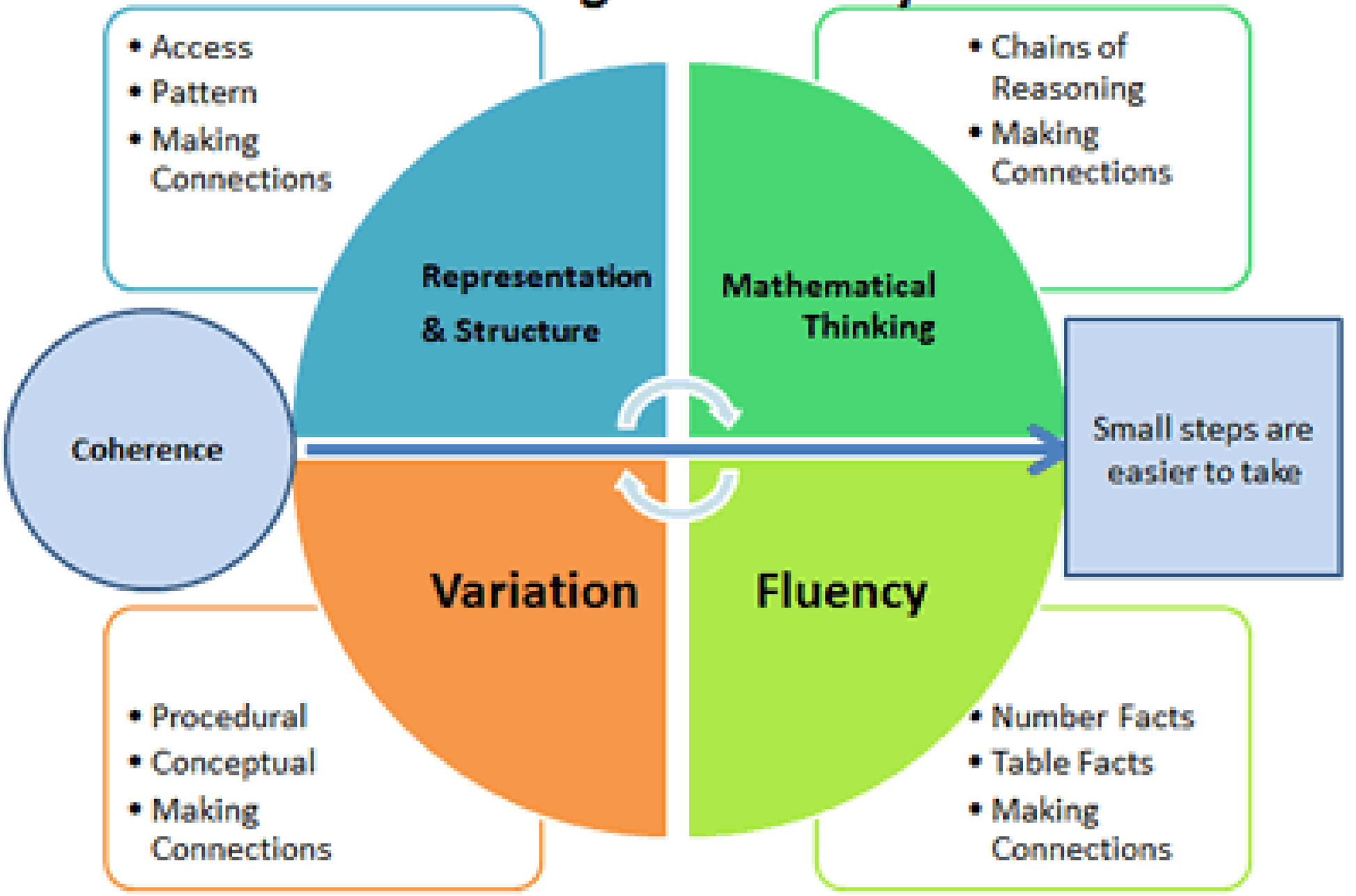
THE ESSENCE OF MATHS TEACHING FOR MASTERY

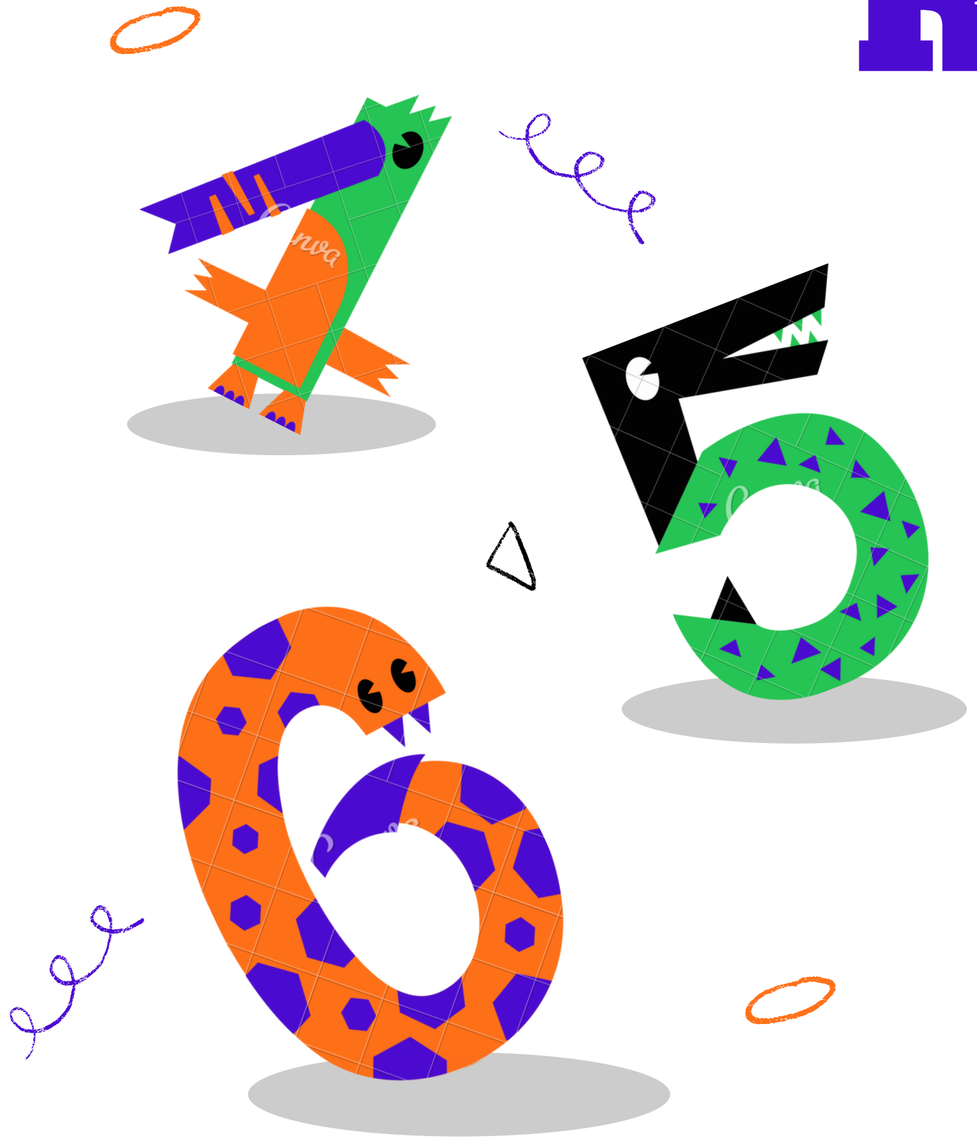
- Maths teaching for mastery rejects the idea that a large proportion of people ‘just can’t do maths’.
- All pupils are encouraged by the belief that by working hard at maths they can succeed.
- Pupils are taught through whole-class interactive teaching, where the focus is on all pupils working together on the same lesson content at the same time, as happens in Shanghai and several other regions that teach maths successfully. This ensures that all can master concepts before moving to the next part of the curriculum sequence, allowing no pupil to be left behind.
- If a pupil fails to grasp a concept or procedure, this is identified quickly and early intervention ensures the pupil is ready to move forward with the whole class in the next lesson.
- Lesson design identifies the new mathematics that is to be taught, the key points, the difficult points and a carefully sequenced journey through the learning. In a typical lesson pupils sit facing the teacher and the teacher leads back and forth interaction, including questioning, short tasks, explanation, demonstration, and discussion.

The Essence of Maths Teaching for Mastery continued...

- Procedural fluency and conceptual understanding are developed in tandem because each supports the development of the other.
- It is recognised that practice is a vital part of learning, but the practice used is intelligent practice that both reinforces pupils' procedural fluency and develops their conceptual understanding.
- Significant time is spent developing deep knowledge of the key ideas that are needed to underpin future learning. The structure and connections within the mathematics are emphasised, so that pupils develop deep learning that can be sustained.
- Key facts such as multiplication tables and addition facts within 10 are learnt to automaticity to avoid cognitive overload in the working memory and enable pupils to focus on new concepts.

Teaching for Mastery





Representation of structure

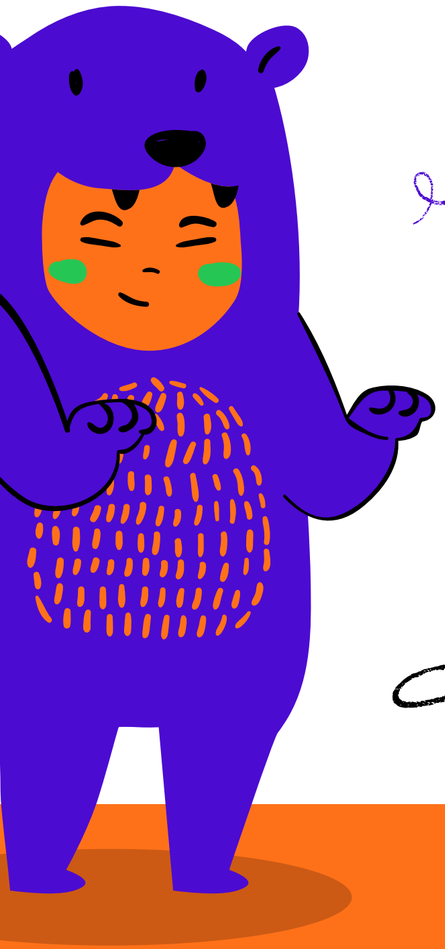
Representations used in lessons expose the mathematical structure being taught, the aim being that students can do the maths without recourse to the representation.

Mathematical Thinking

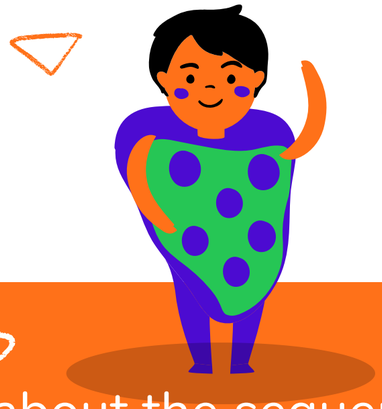
If taught ideas are to be understood deeply, they must not merely be passively received but must be worked on by the student: thought about, reasoned with and discussed with others.

Fluency

Quick and efficient recall of facts and procedures and the flexibility to move between different contexts and representations of mathematics



Variation



It is firstly about how the teacher represents the concept being taught, often in more than one way, to draw attention to critical aspects, and to develop deep and holistic understanding.

Variation is twofold!

It is also about the sequencing of the episodes, activities and exercises used within a lesson and follow up practice, paying attention to what is kept the same and what changes, to connect the mathematics and draw attention to mathematical relationships and structure.

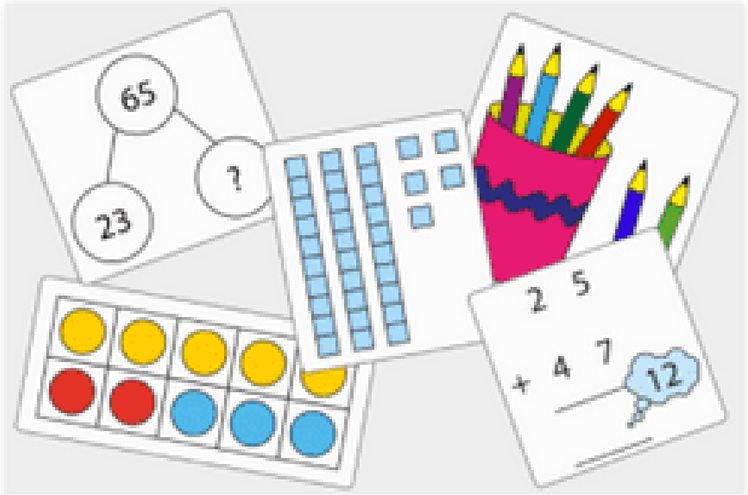
Coherence

Lessons are broken down into small connected steps that gradually unfold the concept, providing access for all children and leading to a generalisation of the concept and the ability to apply the concept to a range of contexts.

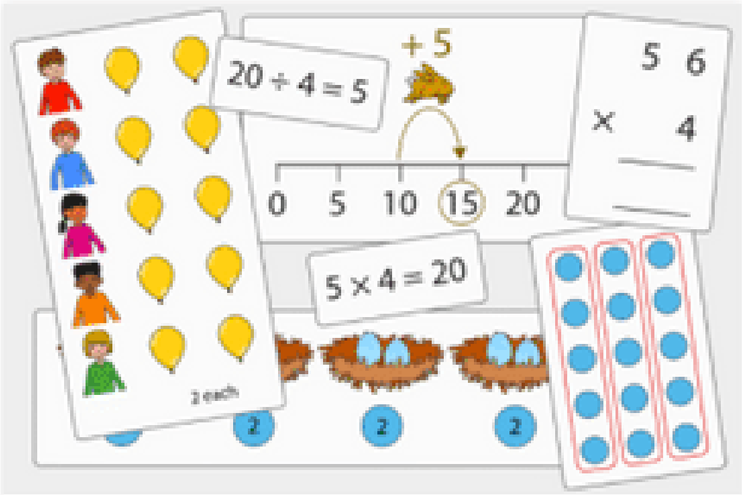
Resources

The NCETM (National Centre for Excellence in the Teaching of Mathematics) have produced materials to support a mastery approach. Longlands began its mastery journey a few years ago using the support of programs and work groups led by SHaW Maths Hub, which is led by NCETM.

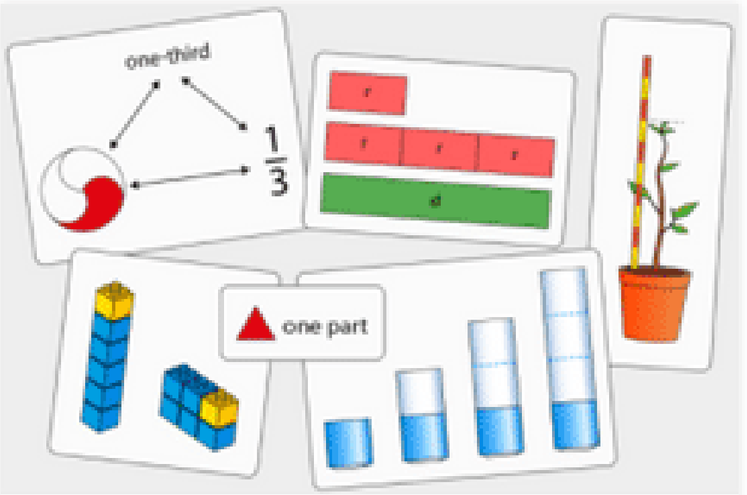
Materials include, mathematics spines and teaching points, assessment materials and ready to progress criteria.



1. Number, Addition and Subtraction



2. Multiplication and Division



3. Fractions

Each spine is composed of a number of segments.

The materials can support teachers to develop their subject and pedagogical knowledge and so help to improve mathematics teaching in combination with other high-quality resources, such as textbooks.

MULTIPLYING WHOLE NUMBERS AND FRACTIONS

Spine 3: Fractions – Topic 3.6

Introduction

Consider multiplication of whole numbers and proper fractions as both repeated addition and scaling. Understand that multiplication of a whole number by a proper fraction results in a smaller number.

Teaching points

- **Teaching point 1**: Repeated addition of proper and improper fractions can be expressed as multiplication of a fraction by a whole number.
- **Teaching point 2**: Repeated addition of a mixed number can be expressed as multiplication of a mixed number by a whole number.
- **Teaching point 3**: Finding a unit fraction of a quantity can be expressed as a multiplication of a whole number by a fraction.
- **Teaching point 4**: A non-unit fraction of a quantity can be calculated by first finding a unit fraction of that quantity.
- **Teaching point 5**: If the size of a non-unit fraction is known, the size of the unit fraction and then the size of the whole can be found.

Phase

PRIMARY KS2 YEAR 4

Related Pages

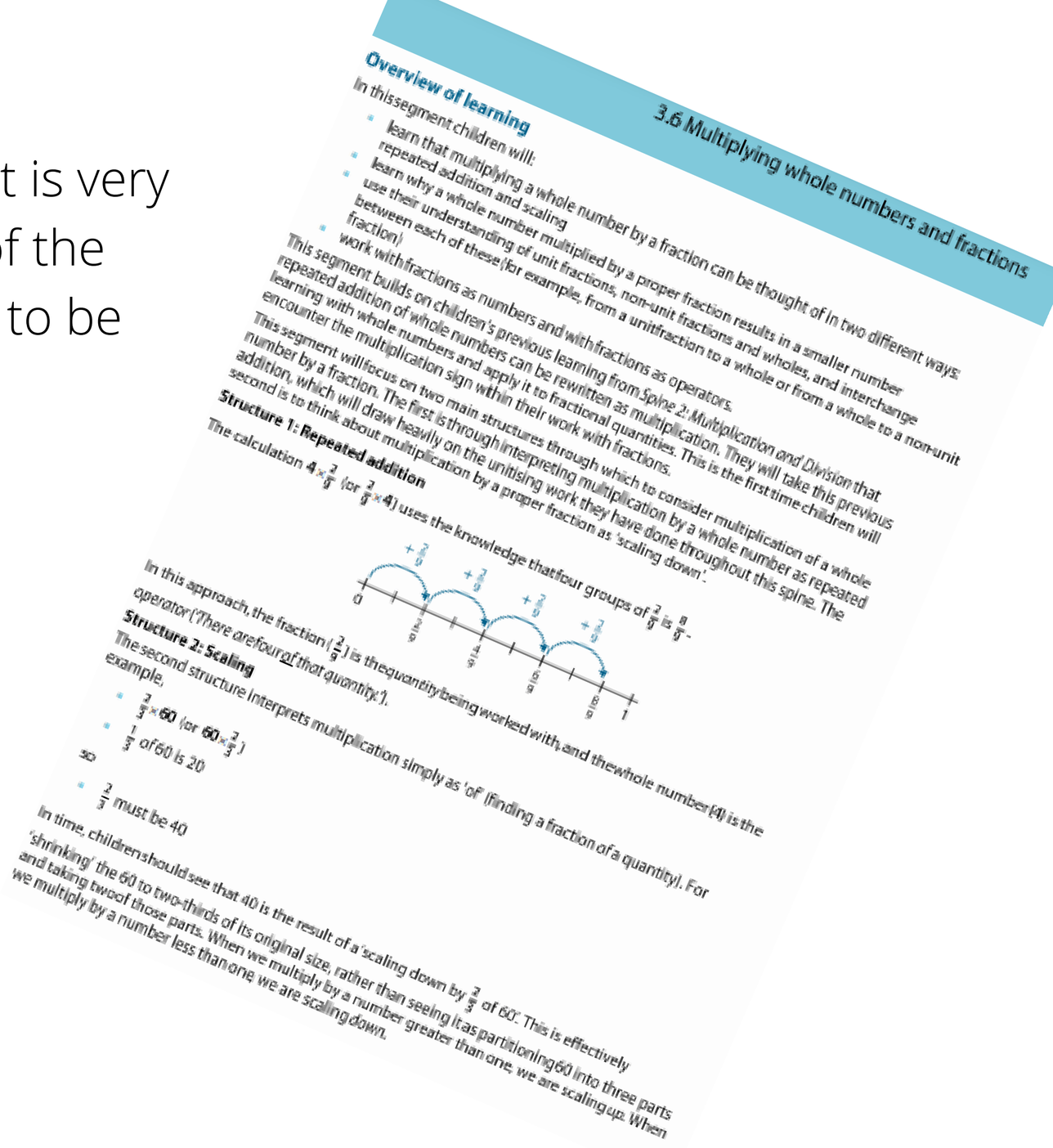
Primary Mastery Professional Development

Fractions

Each spine is broken into segments and each segment has several teaching points.

Each segment has a detailed overview that is very informative and provides information of the pedagogy behind the element of maths to be taught.

Following this is a teacher guide that breaks down each small step, giving ideas and suggestions of how to embed the learning. These teacher guides are full of images and problems that you can use to facilitate the learning in your classroom as well as well-structures stem sentences and generalisations to support learners.



3.6 Multiplying whole numbers and fractions

Overview of learning

In this segment children will:

- learn that multiplying a whole number by a fraction can be thought of in two different ways: repeated addition and scaling
- learn why a whole number multiplied by a proper fraction results in a smaller number
- use their understanding of unit fractions, non-unit fractions and wholes, and interchange between each of these (for example, from a unit fraction to a whole or from a whole to a non-unit fraction)
- work with fractions as numbers and with fractions as operators.

This segment builds on children's previous learning from Spine 2: Multiplication and Division that repeated addition of whole numbers can be rewritten as multiplication. They will take this previous learning with whole numbers and apply it to fractional quantities. This is the first time children will encounter the multiplication sign within their work with fractions.

This segment will focus on two main structures through which to consider multiplication of a whole number by a fraction. The first is through interpreting multiplication by a whole number as repeated addition, which will draw heavily on the unitising work they have done throughout this spine. The second is to think about multiplication by a proper fraction as 'scaling down'.

Structure 1: Repeated addition

The calculation $4 \times \frac{3}{8}$ (or $\frac{3}{8} \times 4$) uses the knowledge that four groups of $\frac{3}{8}$ is $\frac{8}{8}$.

In this approach, the fraction ($\frac{3}{8}$) is the quantity being worked with, and the whole number (4) is the operator ('there are four of that quantity').

Structure 2: Scaling

The second structure interprets multiplication simply as 'of' (finding a fraction of a quantity). For example,

- $\frac{2}{3} \times 60$ (or $60 \times \frac{2}{3}$)
- $\frac{1}{3}$ of 60 is 20

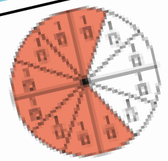
so

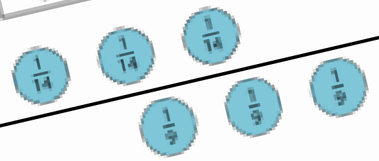
- $\frac{2}{3}$ must be 40

In time, children should see that 40 is the result of a 'scaling down by $\frac{2}{3}$' of 60. This is effectively 'shrinking' the 60 to two-thirds of its original size, rather than seeing it as partitioning 60 into three parts and taking two of those parts. When we multiply by a number greater than one, we are scaling up. When we multiply by a number less than one we are scaling down.

3

Provide children with further examples, such as those provided opposite, and allow them to practise writing multiplication and addition equations where the same unit fraction is repeatedly added.





1:4

Now move toward the abstract and challenge children with missing number problems, without pictorial representations alongside the fractions. Note that in the examples opposite, the fractional part of the multiplicative expression is sometimes placed before the multiplication symbol, and sometimes after.

Where children need additional support, you can encourage them to draw unitising/addition counters to support their learning, as shown in the first example opposite.

$\frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \square \times \frac{1}{9}$

$\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \square \times \frac{1}{9}$

$\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = 5 \times \square$

$\frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{9} \times \square$

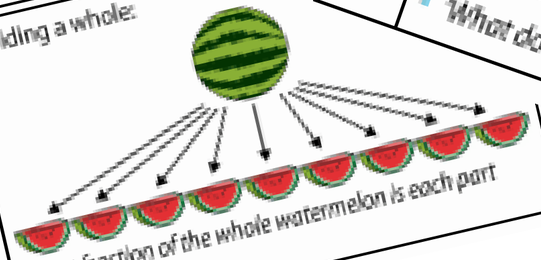
$= 6 \times \frac{1}{9}$

$= \frac{1}{9} \times 5$

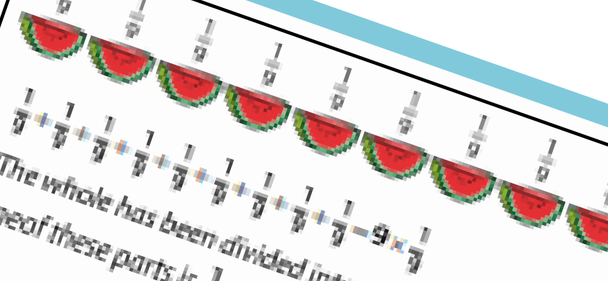
1:5

Once children have mastered this concept, expose them to repeatedly added non-unit fractions. As they are still not required to calculate the solution, this is no more difficult than the previous steps of learning. By introducing it separately, any misconceptions can be addressed as a class, and previous learning applied to find the solution. Again, use familiar

Dividing a whole:



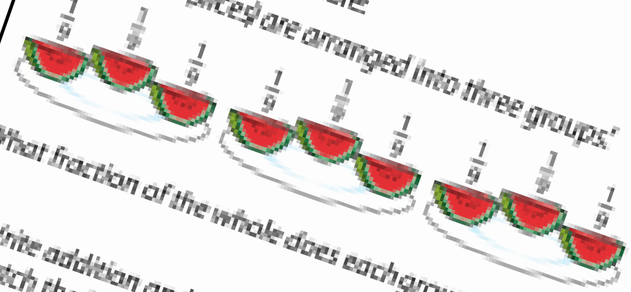
"What fraction of the whole watermelon is each part (slice)?"



- "The whole has been divided into nine equal parts, and one of these parts is $\frac{1}{9}$."

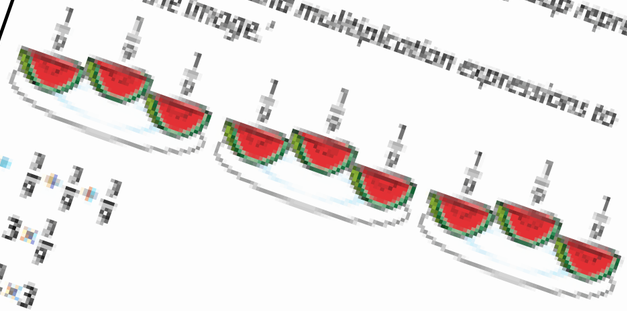
Grouping parts of a whole:

- "The parts (slices) are arranged into three groups."



"What fraction of the whole does each group represent?"

- "Write addition and multiplication expressions to match the image."



- $\frac{1}{9} + \frac{1}{9} + \frac{1}{9}$
- $3 \times \frac{1}{9}$
- $\frac{1}{9} \times 3$

- "What does the '$\frac{1}{9}$' represent?"
- "What does the '3' represent?"

Once children have access this concept, be aware that it may be difficult for some children to understand that multiple objects they can see actually represent a fraction of a whole quantity.

Start with a whole object, such as a watermelon. Split the object into a number of parts and discuss what each of the parts represents. This builds on previous learning. Adapt the stem sentence first encountered in segment 3.2 Unit fractions: identifying, representing and comparing: **"The whole has been divided into ___ equal parts, and ___ of these parts is ___."**

Once it has been established what each part represents, write addition and multiplication expressions, as in the previous step of learning.

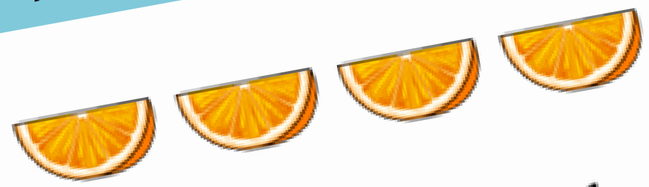
The image can then be manipulated so that more than one fractional part is shown in each group. Keeping the unit fraction labelled on each part and then rearranging the group will help children to understand that these multiple images represent a fraction of a whole.

Write addition and multiplication expressions as before, continuing to link the expressions to the structure:

- "What does the '$\frac{1}{9}$' represent?"
- "What does the '3' represent?"

The teaching guides are accompanied by presentations that are very succinct in revealing the structures behind the mathematical concepts bwing taught. These can be use to provide valuable visuals for out learners.

6 Multiplying whole numbers and fractions Step 1:2



$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

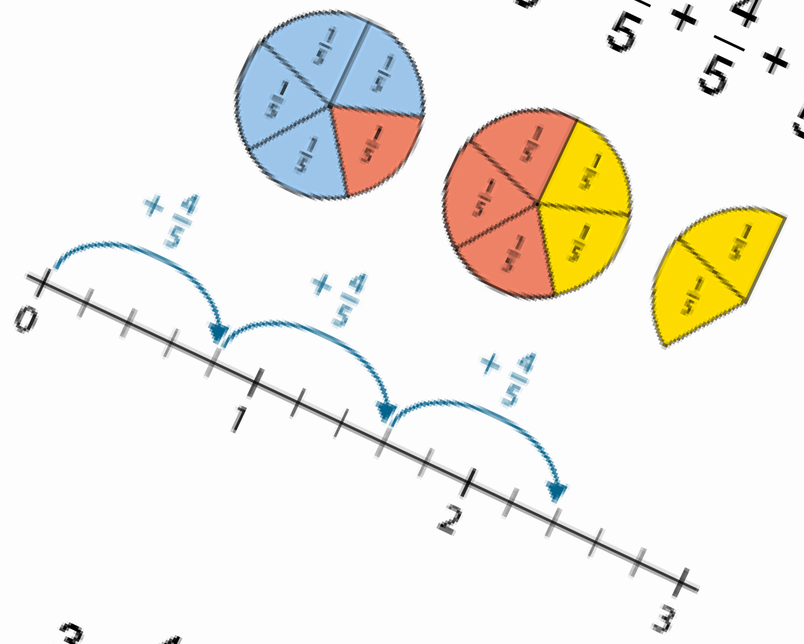
$$4 \times \frac{1}{8}$$

$$\frac{1}{8} \times 4$$

$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 4 \times \frac{1}{8} = \frac{1}{8} \times 4$$

3.6 Multiplying whole numbers and fracti

$$3 \times \frac{4}{5} = \frac{4}{5} + \frac{4}{5} + \frac{4}{5}$$



$$3 \times \frac{4}{5} = \frac{12}{5} = 2\frac{2}{5}$$

What a lesson looks like.

Lessons will commonly be taught using a ‘ping pong’ style approach, so called because the teacher orchestrates a continual back-and-forth dialogue with the children, using questions, short tasks, explanations, demonstrations, and discussions. This enables the teacher to vary the pace and direction of the lesson if necessary, and to continuously monitor the progress of the class.

Any children who struggle with a concept are identified in the lesson and immediate intervention or extra support is put in place so that they are able to keep up with the rest of the class in the next lesson.

What have we done to personalise our maths curriculum?

1. Used the National curriculum and identidied areas that were not included in the spines. This highlighted for teachers the areas that needed to be covered at other points in the year or though other subjects. E.g. time,

2. Looked at the elements thrat were not in the year groups as suggested by the NC and decided where the objectives would be covered in our own curriculum.

3. Noted how long each spine took to teach each year that it has been so as to form a basis for a guide for future teachers.

Assessment

1. Lessons are structured so that assessment of learning is continual. Any misconceptions are dealt with immediately but idealy already planned for and addressed in teaching.

2. Sticky quizzes include maths questions distanced from the teaching of a specific concept to establish how much has been retained and can be applied.

3. A range of assessments have been analysed and implemented termly to provide summative data. PUMA tests are taken as well as ast end of key stage papers for years 2 and 6. Recently developed assessments that mirror the domains and question level of past papers are also available for years 3-5 all year and to year 1 in th summer term.

Professional Development

Nearly all of our teaching staff have accessed a 'Developing Teaching for Mastery' programme. As a Mastery Specialist Teacher, colleagues within school can join my own work group or others.

Our Year 2 teacher participated in the EEF Reasoning project and continues to make good use of the research and resources every year.

Moving forward, teachers new to EYFS are joingng 'Early Years - Specialist Knowledge for Teaching Mathematics - Number Patterns and Structures' work group. The teacher new to Year 1 is joinging the year 1 Specialist knowledge work group.

EYFS, year 1 and year 2 teachers are also enrolled on 'Mastering Number'. This project aims to secure firm foundations in the development of good number sense for all children from Reception through to Year 1 and Year 2. The aim over time is that children will leave KS1 with fluency in calculation and a confidence and flexibility with number. new to all programme to

Next Steps

1. Recent monitoring highlighted the need to reignite the hands-on elements of mathematics that COVID has not allowed as soon as we are able to do so.

2. Consider how depth of thinking can be noted in books to distinguish between the levels of understanding and ability to apply withon a class.

3. Distanced assessment is required more frequently to fully establish a clear picture of the retention of knowledge and skills away from learning.